Lesson 20. The Points-After-Touchdown Problem

1 The problem

- In an NFL football game, after scoring a touchdown, a team is given the option to try for:
    - a **1-point conversion**: 1 extra point by a field goal from the 15-yard line, or
    - a **2-point conversion**: 2 extra points by advancing the ball into the end zone from the 2-yard line
- Whether to "go for 2" is a classic debate – a few somewhat recent discussions on the topic:
    - https://theringer.com/nfl-two-point-conversions-pittsburgh-steelers-mike-tomlin-65d47282d853
    - https://fivethirtyeight.com/features/more-nfl-teams-are-going-for-two-just-as-they-should-be/
- Adding to the debate: in 2015, 1-point attempts were moved from the 2-yard line to the 15-yard line
- Conversion success rates from the past 4 regular seasons (from http://www.pro-football-reference.com/):

	2014	2015	2016	2017
1-point conversion success rate	0.993	0.942	0.936	0.940
2-point conversion success rate	0.483	0.479	0.486	0.451

- Based on the current score and time remaining, should a team "go for 1" or "go for 2" in order to maximize the probability that it wins the game?
- How does the 2015 rule change affect a team's optimal conversion strategy?
- Let's try to answer these questions by modeling this problem as a stochastic dynamic program
- We will be roughly following this paper:

    H. Sackrowitz (2000). Refining the point(s)-after-touchdown decision. *Chance* **13**(3): 29-34.

2 Data

- Two teams: A and B
  - Assume that we (the decision-makers) are Team A
- Suppose we have the following data:

$$
\begin{aligned}
T &= \text{total number of possessions} \\
p_n &= \Pr\{\text{1-pt. conv. successful for Team } n \mid \text{1-pt. conv. attempted by Team } n\} && \text{for } n = \text{A, B} \\
q_n &= \Pr\{\text{2-pt. conv. successful for Team } n \mid \text{2-pt. conv. attempted by Team } n\} && \text{for } n = \text{A, B} \\
b_1 &= \Pr\{\text{1-pt. conv. attempted by Team B}\} \\
b_2 &= \Pr\{\text{2-pt. conv. attempted by Team B}\} \\
t_n &= \Pr\{\text{TD by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
g_n &= \Pr\{\text{FG by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
z_n &= \Pr\{\text{no score by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
r &= \Pr\{\text{Team A wins in overtime}\}
\end{aligned}
$$

- What is the relationship between b_1 and b_2?

$$b_1 + b_2 = 1$$

- What is the relationship between t_n, g_n and z_n?

$$t_n + g_n + z_n = 1$$

- What is the probability that Team B scores 0 after a touchdown?

$$1 - (b_1 p_B + b_2 q_B)$$

b_1: Team B goes for 1?
p_B: Team B makes 1?
b_2: Team B goes for 2?
q_B: Team B makes 2?

3 The stochastic DP

- Stages:

$$\begin{aligned} t = 0, 1, \dots, T-1 &\quad \leftrightarrow \quad \text{end of possession } t \\ t = T &\quad \leftrightarrow \quad \text{end of game} \end{aligned}$$

  - For our purposes, a possession ends when a team scores (TD or FG), or loses possession without scoring

- States:

$$(n, k, d) \quad \leftrightarrow \quad \begin{array}{ll} \text{Team } n\text{'s possession just ended} & \text{for } n \in \{\text{A}, \text{B}\} \\ \text{Team } n \text{ just scored } k \text{ points} & \text{for } k \in \{0, 3, 6\} \\ \text{Team A is ahead by } d \text{ points} & \text{for } d \in \{\dots, -1, 0, 1, \dots, \} \end{array}$$

- Value-to-go function:

$$f_t(n, k, d) = \text{maximum probability that Team A wins when in state } (n, k, d) \text{ at the end of possession } t$$
$$\text{for } n \in \{\text{A}, \text{B}\}, k \in \{0, 3, 6\}, d \in \{\dots, -1, 0, 1, \dots\}$$

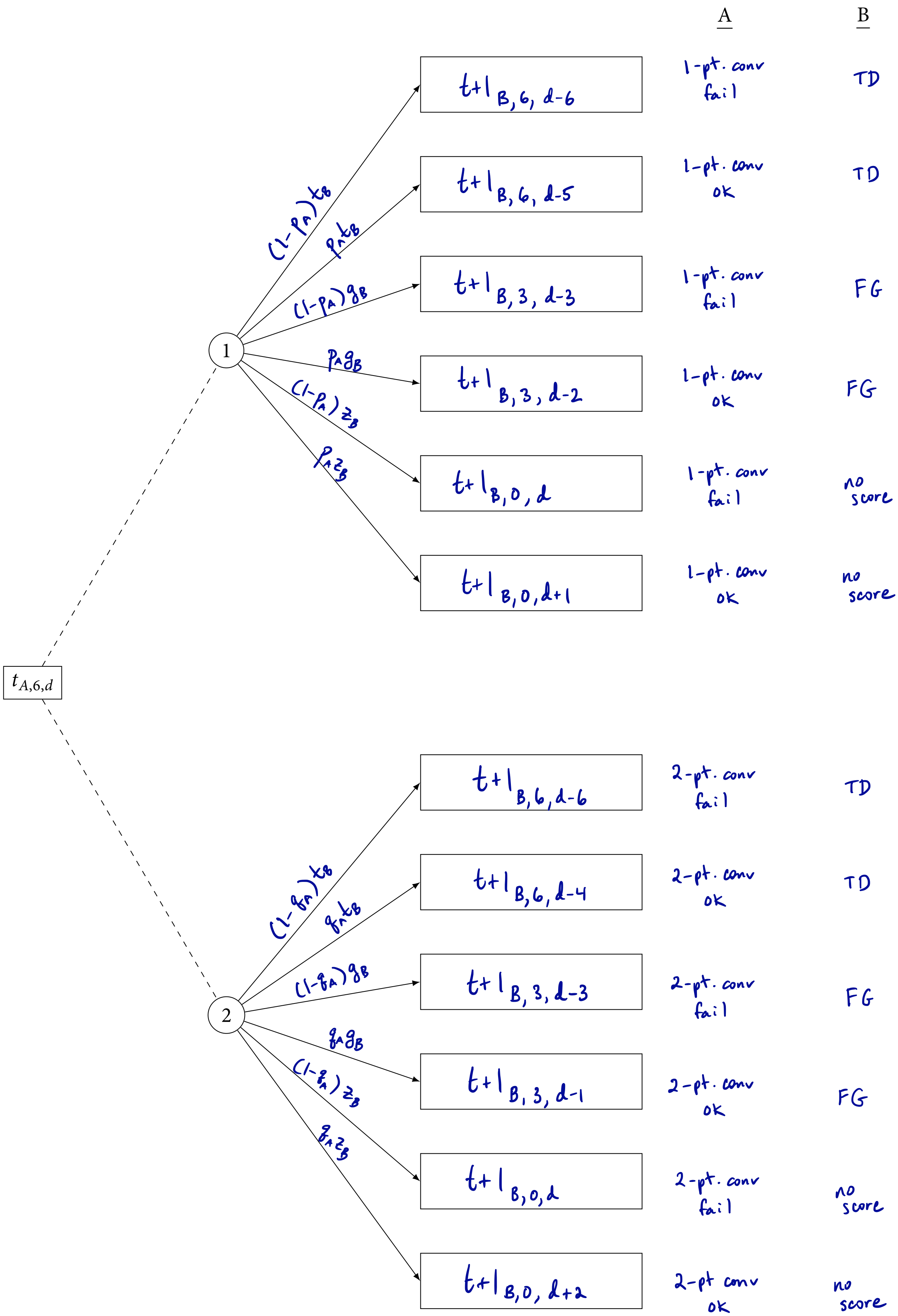
- Allowable decisions x_t at stage t and state (n, k, d):

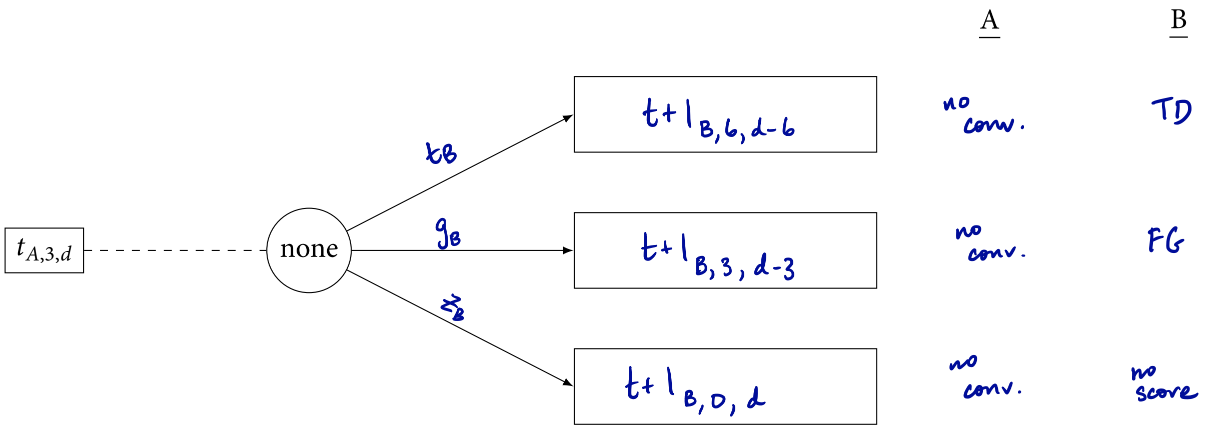
$$\begin{array}{ll} x_t \in \{1, 2\} & \text{if } n = \text{A and } k = 6 \\ x_t = \text{none} & \text{if } n = \text{A and } k \in \{0, 3\} \\ x_t = \text{none} & \text{if } n = \text{B and } k \in \{0, 3, 6\} \end{array}$$

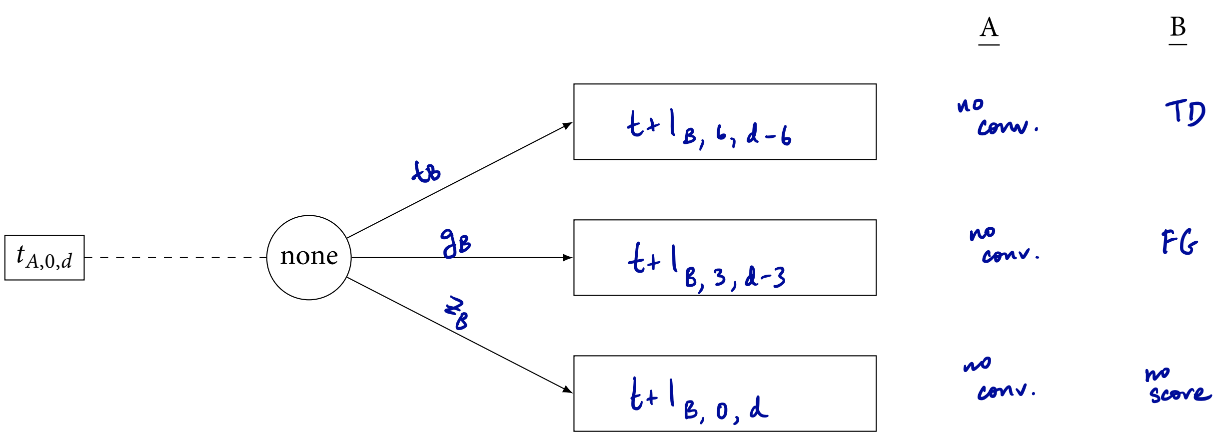
- We need to consider transitions from the following states:

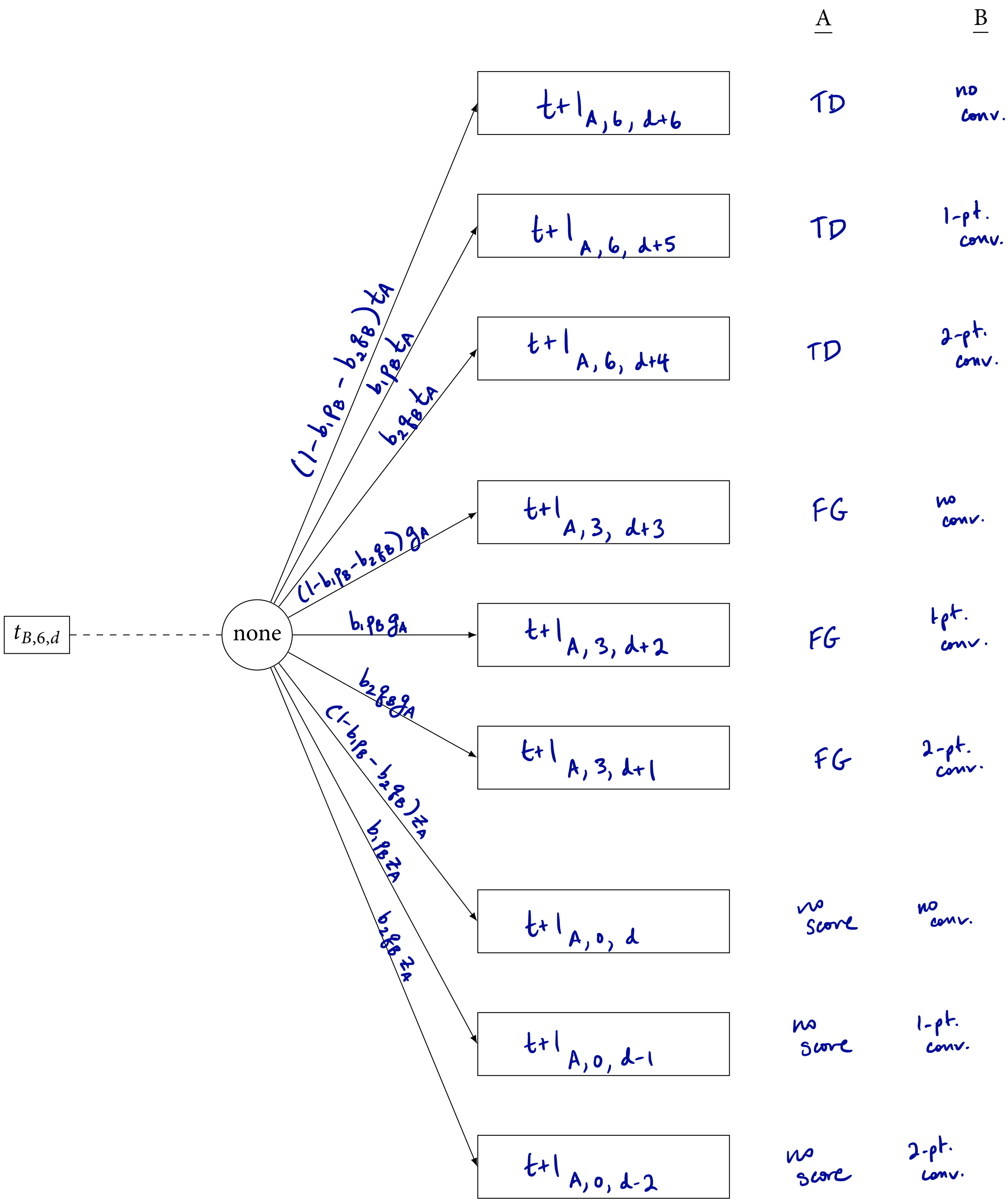
$$\begin{array}{lll} (\text{A}, 6, d) & (\text{A}, 3, d) & (\text{A}, 0, d) \\ (\text{B}, 6, d) & (\text{B}, 3, d) & (\text{B}, 0, d) \end{array} \qquad \text{for all } d$$

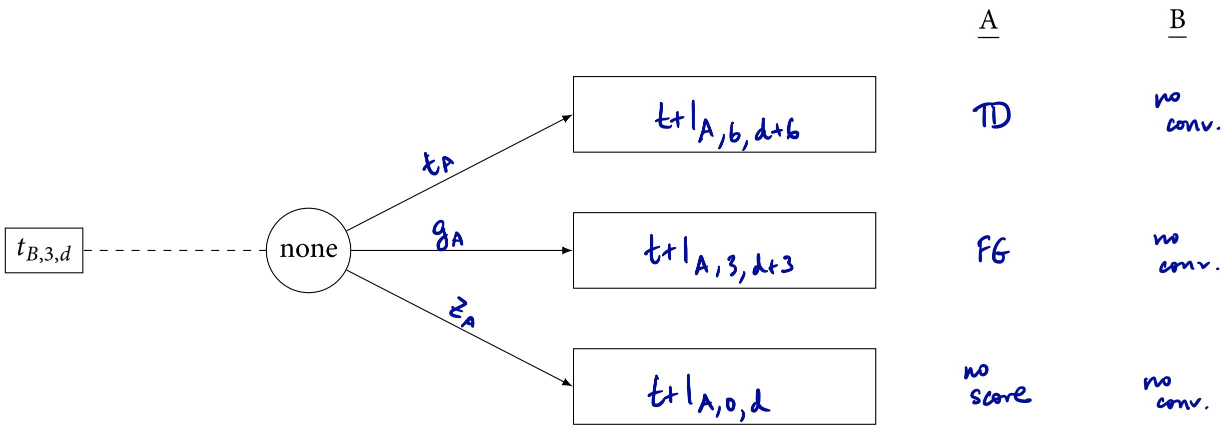
- Since our objective is to maximize the probability of winning, we set all the contributions in stages $t = 0, 1, \dots, T-1$ to 0, just like in the investment problem in Lesson 18

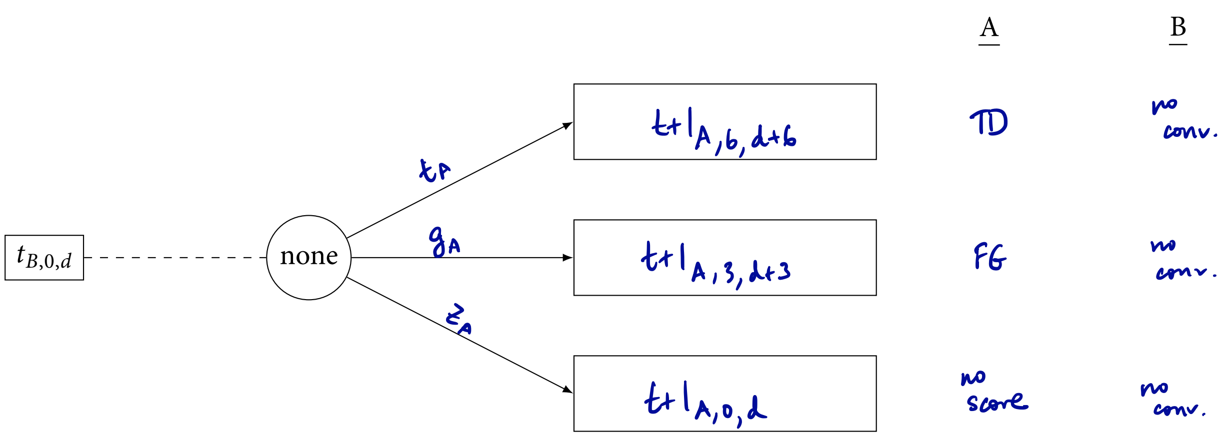

A
B
$t_{A,6,d}$
1
$(1-p_A)t_B$
$p_A t_B$
$(1-p_A)g_B$
$p_A g_B$
$(1-p_A)z_B$
$p_A z_B$
$t+1_{B,6,d-6}$
1-pt. conv fail
TD
$t+1_{B,6,d-5}$
1-pt. conv ok
TD
$t+1_{B,3,d-3}$
1-pt. conv fail
FG
$t+1_{B,3,d-2}$
1-pt. conv ok
FG
$t+1_{B,0,d}$
1-pt. conv fail
no score
$t+1_{B,0,d+1}$
1-pt. conv ok
no score
2
$(1-q_A)t_B$
$q_A t_B$
$(1-q_A)g_B$
$q_A g_B$
$(1-q_A)z_B$
$q_A z_B$
$t+1_{B,6,d-6}$
2-pt. conv fail
TD
$t+1_{B,6,d-4}$
2-pt. conv ok
TD
$t+1_{B,3,d-3}$
2-pt. conv fail
FG
$t+1_{B,3,d-1}$
2-pt. conv ok
FG
$t+1_{B,0,d}$
2-pt. conv fail
no score
$t+1_{B,0,d+2}$
2-pt conv ok
no score


A
B
$t_{A,3,d}$
none
tB
gB
zB
t+1 B,6,d-6
t+1 B,3,d-3
t+1 B,0,d
no conv.
no conv.
no conv.
TD
FG
no score

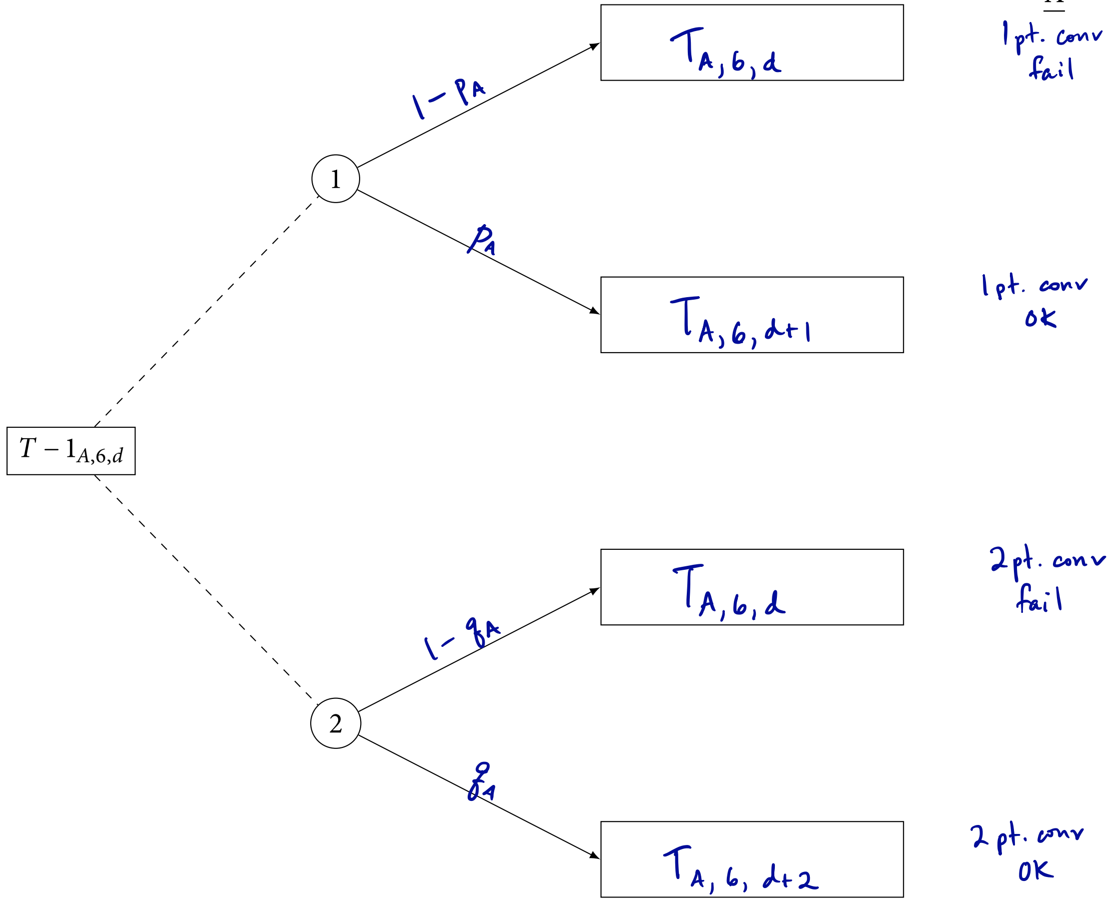

A
B
$t_{A,0,d}$
none
tB
gB
zB
t+1 B,6,d-6
t+1 B,3,d-3
t+1 B,0,d
no conv.
no conv.
no conv.
TD
FG
no score


A
B
$t_{B,6,d}$
none
$(1-b_1p_B-b_2q_B)t_A$
$b_1p_Bt_A$
$b_2q_Bt_A$
$(1-b_1p_B-b_2q_B)g_A$
$b_1p_Bg_A$
$b_2q_Bg_A$
$(1-b_1p_B-b_2q_B)z_A$
$b_1p_Bz_A$
$b_2q_Bz_A$
$t+1_{A,6,d+6}$
TD
no conv.
$t+1_{A,6,d+5}$
TD
1-pt. conv.
$t+1_{A,6,d+4}$
TD
2-pt. conv.
$t+1_{A,3,d+3}$
FG
no conv.
$t+1_{A,3,d+2}$
FG
1-pt. conv.
$t+1_{A,3,d+1}$
FG
2-pt. conv.
$t+1_{A,0,d}$
no score
no conv.
$t+1_{A,0,d-1}$
no score
1-pt. conv.
$t+1_{A,0,d-2}$
no score
2-pt. conv.

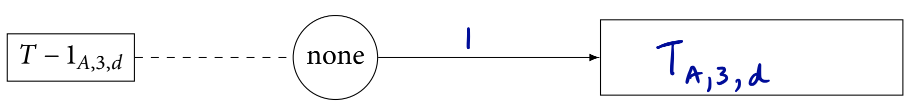

A
B
$t_{B,3,d}$
none
tA
gA
zA
t+1 A,6,d+6
t+1 A,3,d+3
t+1 A,0,d
TD
FG
no score
no conv.
no conv.
no conv.


A
B
$t_{B,0,d}$
none
tA
gA
zA
t+1 A,6,d+6
t+1 A,3,d+3
t+1 A,0,d
TD
FG
no score
no conv.
no conv.
no conv.

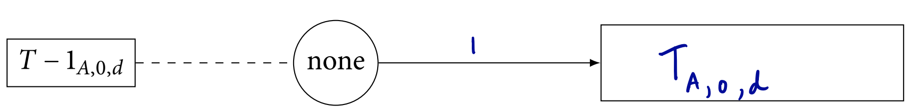
A

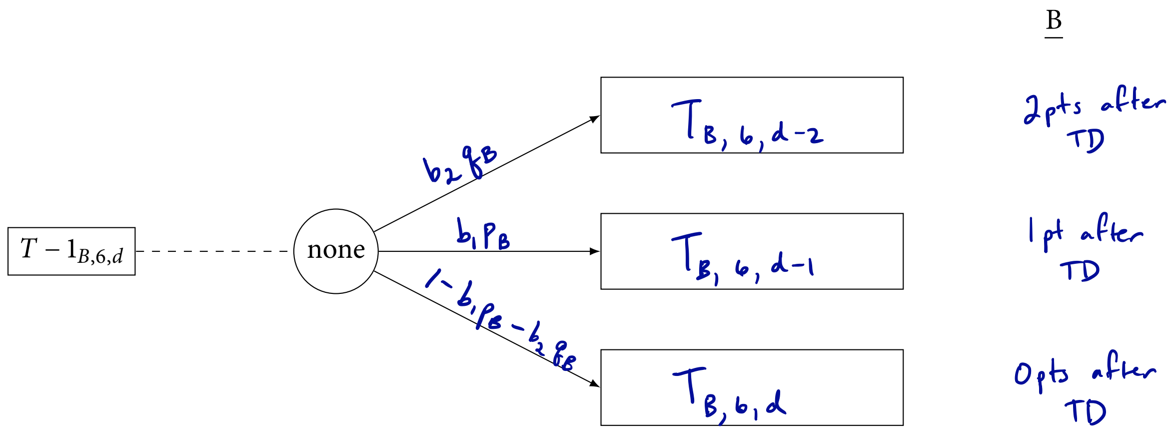

$T-1_{A,6,d}$
1
2
$1-p_A$
p_A
$1-q_A$
q_A
$T_{A,6,d}$
$T_{A,6,d+1}$
$T_{A,6,d}$
$T_{A,6,d+2}$
1pt. conv fail
1pt. conv OK
2pt. conv fail
2pt. conv OK

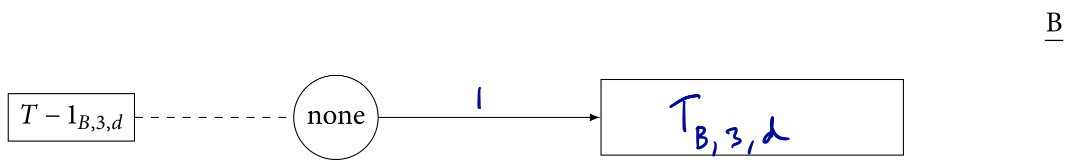
A

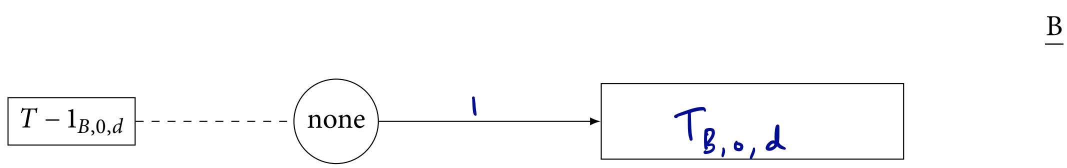

$T-1_{A,3,d}$
none
1
$T_{A,3,d}$

A


$T-1_{A,0,d}$
none
1
$T_{A,0,d}$


B
T − 1B,6,d
none
b2 qB
b1 pB
1 − b1 pB − b2 qB
TB,6,d−2
TB,6,d−1
TB,6,d
2pts after TD
1pt after TD
0pts after TD


B
T − 1B,3,d
none
1
TB,3,d


B
T − 1B,0,d
none
1
TB,0,d